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SEISMIC RESPONSE OF SLIDING STRUCTURES TO BIDIRECTIONAL EARTHQUAKE EXCITATION

R. S. JANGID

Department of Civil Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai-400 076 India

SUMMARY

Seismic response of a one-storey structure with sliding support to bidirectional (i.e. two horizontal components) earthquake ground motion is investigated. Frictional forces, which are mobilized at the sliding support, are assumed to have ideal Coulomb-friction characteristics. Coupling effects due to circular interaction between the frictional forces are incorporated in the governing equations of motion. Effects of bidirectional interaction of frictional forces on the response are investigated by comparing the response to two-component excitation with the corresponding response produced by the application of single-component excitations in each direction independently. It is observed that the response of the sliding structure is influenced significantly by the bidirectional interaction of frictional forces. Further, it is shown that the design sliding displacement may be underestimated if the bidirectional interaction of frictional forces is neglected and the sliding structures are designed merely on the basis of single-component excitation.

KEY WORDS: sliding structure; pure-friction; earthquake; bidirectional interaction

INTRODUCTION

Consider an elastic one-storey structure with a sliding support between the base mass and the foundation as shown in Figure 1. This model of sliding structure has been widely studied under unidirectional support motion.[1–4] The sliding support is isotropic and the frictional forces mobilized at the sliding support have the ideal Coulomb-friction characteristics (i.e. the coefficient of friction of the sliding support remains constant and independent of the pressure and velocity). Further, the superstructure is symmetric with respect to two orthogonal directions (referred to as x- and y-directions). As a result, there is no torsional coupling with lateral movement of the system. Therefore, the system has four degrees of freedom (DOF) under the bidirectional horizontal earthquake ground motion viz. displacements of superstructure (x_S and y_S) relative to the base mass and the displacement of base mass (x_b and y_b) relative to the ground in two orthogonal x- and y-directions, respectively. Governing equations of motion can be derived as

$$m_s\ddot{x}_s + c_x\dot{x}_s + k_x x_s = -m_s(\ddot{x}_g + \ddot{x}_b) \tag{1a}$$

$$m_s\ddot{y}_s + c_y\dot{y}_s + k_y y_s = -m_s(\ddot{y}_g + \ddot{y}_b) \tag{1b}$$

$$m_b\ddot{x}_b + F_x - c_x\dot{x}_s - k_x x_s = -m_b\ddot{x}_g \tag{2a}$$

$$m_b\ddot{y}_b + F_y - c_y\dot{y}_s - k_y y_s = -m_b\ddot{y}_g \tag{2b}$$

where m_s is the mass of the superstructure, c_x and c_y are the damping of the superstructure in x- and y-direction, respectively, k_x and k_y are the stiffness of superstructure of the superstructure in x- and y-directions, respectively, m_b is the mass of the base raft, and $\ddot{x}_g$ and $\ddot{y}_g$ and F_x and F_y are the earthquake ground accelerations and the frictional forces at the sliding support, respectively, in the x- and y-directions of the system. The limiting value of the frictional force F_S which the sliding support can be subjected is expressed as

$$F_s = \mu(m_s + m_b)g \tag{3}$$

where μ is the friction coefficient of sliding interface and g is the acceleration due to gravity.

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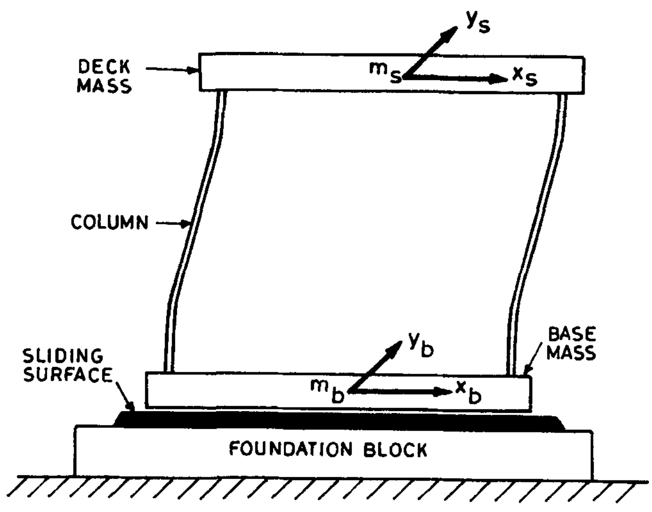

Figure 1. One-storey system with sliding support at the base

Criteria for sliding and non-sliding phases

In a non-sliding phase ($\ddot{x}_b = \ddot{y}_b = 0$ and $\dot{x}_b = \dot{y}_b = 0$) the resultant of the frictional forces mobilized at the sliding interface is less than the limiting frictional force, i.e.

$$\sqrt{F_x^2 + F_y^2} < F_s \tag{4a}$$

The system will start sliding ($\ddot{x}_b \neq \ddot{y}_b \neq 0$ and $\dot{x}_b \neq \dot{y}_b \neq 0$) as soon as this resultant exceeds the limiting frictional force. Thus, the sliding phase of the system will take place if

$$\sqrt{F_x^2 + F_y^2} = F_s \tag{4b}$$

Note that equation (4b) depicts a circular interaction curve between the frictional forces mobilized at the sliding support as shown in Figure 2(a). Because of the interaction between the frictional forces, the governing equations of motion of the sliding structures in two orthogonal directions are coupled. Therefore, it is interesting to investigate the effects of bidirectional interaction of frictional forces on the response of sliding structures under earthquake ground motion. However, this interaction effect is ignored when the structural system is modelled as a 2-D system.

During the non-sliding phase (till the inequality (4a) holds good), response of the system can be obtained by considering single-DOF system in two orthogonal directions independently. The failure of inequality (4a) indicates the occurrence of sliding phase and the equations of motion of base mass should also be integrated. Equations of motion in the sliding phase need to be solved in the incremental form due to (i) dependence of the frictional forces on the relative velocities of the base mass, and (ii) circular interaction between the mobilized frictional forces (see Figure 2(a)). By assuming the linear variation of acceleration over a small time interval δt, equations (1) and (2) can be expressed as

$$[C_{\mathrm{eff}}]\{\dot{X}^{t+\delta t}\} = \{P_{\mathrm{eff}}\} + \{\delta F\} \tag{5}$$

where $[C_{\mathrm{eff}}]$ is the matrix of size (4×4), $\{\dot{X}^{t+\delta t}\} = \{\dot{x}_s^{t+\delta t}, \dot{y}_s^{t+\delta t}, \dot{x}_b^{t+\delta t}, \dot{y}_b^{t+\delta t}\}^{\mathrm{T}}$ is the velocity vector at time $t + \delta t$, $\{P_{\mathrm{eff}}\}$ is the effective excitation force vector, and $\{\delta F\} = \{0, 0, -\delta F_x, -\delta F_y\}^{\mathrm{T}}$ is the incremental frictional force vector. T denotes the transpose and the superscript denotes the time.

The incremental frictional forces can be determined from Figure 2(b). At time t, the frictional forces are at point A (shown in the figure) on the interaction curve and move to point B at time $t + \delta t$. Since the frictional forces oppose the motion of the system, the angles θ^t and $\theta^{t+\delta t}$ will provide the direction of sliding at time

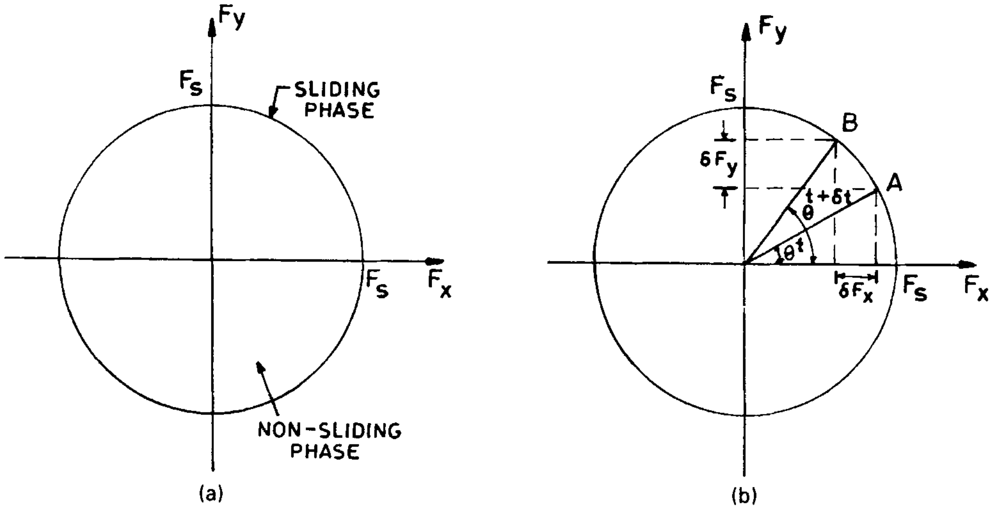

Figure 2. The interaction between frictional forces and the incremental frictional forces during sliding phase

t and $t + \delta t$, respectively. Further, the direction of sliding $\theta^{t+\delta t}$ is equal to $\tan^{-1}(\dot{y}_b^{t+\delta t}/\dot{x}_b^{t+\delta t})$. Hence, the incremental frictional forces can be obtained from Figure 2(b) as

$$\delta F_x = F_s \frac{\dot{x}_b^{t+\delta t}}{\sqrt{(\dot{x}_b^{t+\delta t})^2 + (\dot{y}_b^{t+\delta t})^2}} - F_x^t \tag{6a}$$

$$\delta F_y = F_s \frac{\dot{y}_b^{t+\delta t}}{\sqrt{(\dot{x}_b^{t+\delta t})^2 + (\dot{y}_b^{t+\delta t})^2}} - F_y^t \tag{6b}$$

Note that the incremental matrix equation (5) is non-linear because of the interaction of the frictional forces in two orthogonal directions. However, this non-linearity is circumvented by employing the iterations in each time step.

NUMERICAL STUDY

Response quantities of interest for the system under consideration are the absolute acceleration of the superstructure (in x-direction, $\ddot{x}_a = \ddot{x}_s + \ddot{x}_b + \ddot{x}_g$ and in y-direction $\ddot{y}_a = \ddot{y}_s + \ddot{y}_b + \ddot{y}_g$) and the relative sliding base displacement (x_b and y_b). The absolute acceleration is directly proportional to the forces exerted in the superstructure due to earthquake ground motion. On the other hand, the relative sliding base displacement is crucial from the design point of view of sliding system. The N00E component of the El-Centro 1940 earthquake is applied in the x-direction (other orthogonal component is applied in the y-direction) of the system. This response of the system is referred to as the response to *two-component* excitation. The response of the system is also obtained for the same components acting independently in each direction (referred to as *single-component*) in which there is no interaction between frictional forces in two orthogonal directions. In the present study, the time period of the superstructure ($T_s = 2\pi\sqrt{m_s/k_x} = 2\pi\sqrt{m_s/k_y}$) as a fixed base is kept same in two orthogonal directions. The damping ratio of the superstructure is taken as 5 per cent of the critical in both directions. The friction coefficient of the sliding support, $\mu = 0{\cdot}1$ and the mass ratio m_b/m_s is taken as unity. The response of the sliding structure is found to be very sensitive to the starting times of sliding and non-sliding phases implying that the digitized time interval, δt should be very small. Thus, $\delta t = 0{\cdot}02/100$ is employed for both sliding and non-sliding phases. Still smaller $\delta t = 0.02/1000$ has been used in the neighbourhood of transition of phases. Further, the sliding velocity less than 1×10^{-8} m/s is assumed to be zero for checking the transition from sliding to non-sliding phase.

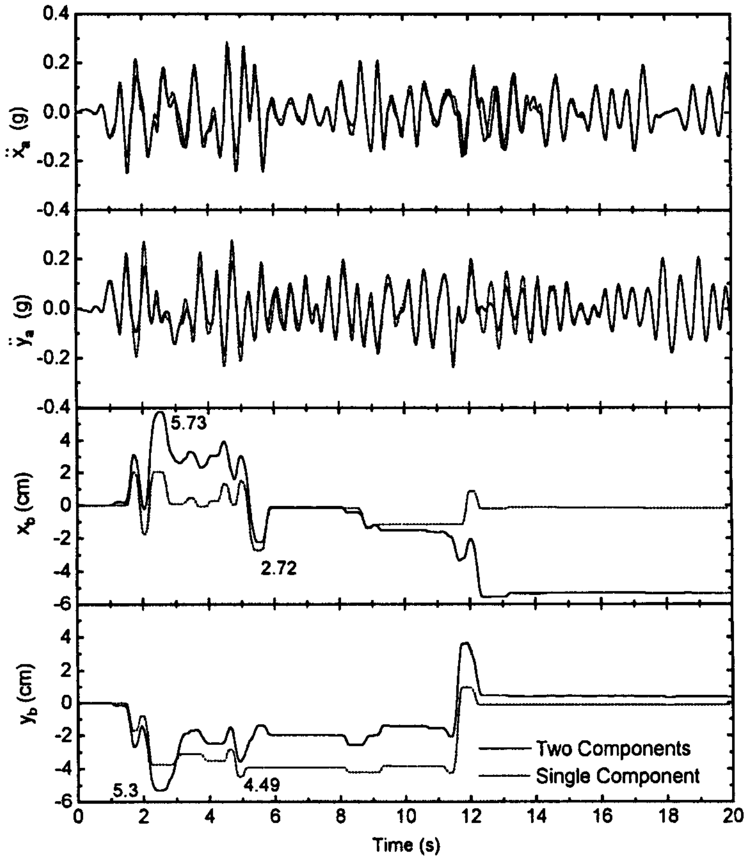

Figure 3. Time history of the absolute acceleration of the superstructure and base displacement to the El-Centro 1940 earthquake motion ($T_s = 0.5$ s)

In Figure 3, the time variation of the absolute acceleration of superstructure ($\ddot{x}_a$ and $\ddot{y}_a$) and the sliding base displacement in x- and y-directions are plotted for both single and two components of El-Centro 1940 earthquake ground motion. The figure indicates that the nature of the variation of the absolute acceleration is almost the same for both cases. The absolute acceleration of the superstructure is relatively less for two-component ground motion as compared to those with single-component ground motion. On the other hand, there is a significant difference in the base displacement for two-component and single-component earthquake ground motions. The base displacements are relatively higher for the former case. This is due to the fact that for the two-component earthquake ground motion, the system starts sliding at a relatively lower value of the frictional forces mobilized at the sliding support (refer the interaction equation (4b)); as a result, there is more sliding displacement. Thus, the sliding base displacements may be underestimated if the two components of earthquake ground motion are not considered simultaneously for designing the sliding support.

Figure 4 shows the variation of the resultant peak absolute acceleration of the superstructure (i.e. $\sqrt{(\ddot{x}_a)^2_{max} + (\ddot{y}_a)^2_{max}}$) against T_s. The figure indicates that the absolute acceleration of the superstructure is less for two-component ground motion in comparison with that for single-component earthquake ground motion. Note that $T_s = 0$ is the case of the rigid structure with sliding interface. The absolute acceleration spectra of the superstructure without sliding support (referred to as no slip) are also shown in order to study the effectiveness of the sliding support. The figure indicates clearly that the sliding support is quite effective in reducing the earthquake response of the superstructure. Further, the absolute acceleration of the system with sliding base is less sensitive to the time period of the superstructure in comparison with fixed base system.

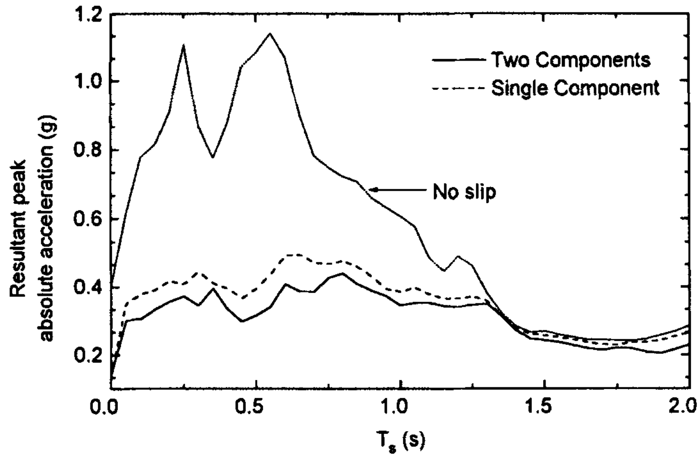

Figure 4. Plot of the resultant peak absolute acceleration of the superstructure against the time period of superstructure to the El-Centro 1940 earthquake motion

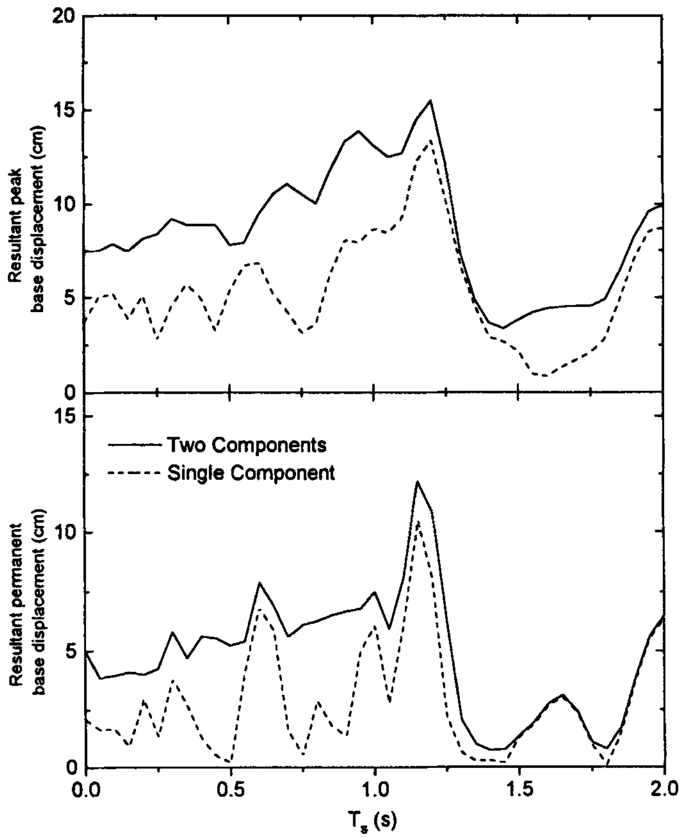

Figure 5. Plot of the resultant peak base displacement and permanent base displacement against the time period of superstructure to the El-Centro 1940 earthquake motion

In Figure 5, the variation of the resultant peak sliding base displacement and the resultant permanent displacement is plotted against the T_s. The figure clearly shows that both the peak as well as permanent base displacements are significantly higher for two-component excitation in comparison with the single-

component excitation. Thus, there is a need to consider the bidirectional interaction effects of frictional forces on the response. Note that the similar effects of bidirectional interaction of frictional forces for structures isolated by Teflon sliding bearing were observed by Mokha *et al.*[5] and the same are further confirmed in the present study for pure-friction sliding structures.

CONCLUSIONS

Seismic response of the sliding structures to two horizontal components of earthquake ground motion is investigated by both considering as well as ignoring the interaction between the frictional forces mobilized at the sliding support in two orthogonal directions. Numerical results show that the bidirectional excitation increases the sliding base displacement and decreases the absolute acceleration of the superstructure. Thus, the design sliding displacement may be underestimated if the bidirectional interaction of frictional forces is neglected and the sliding structures are designed merely on the basis of single-component excitation.

REFERENCES

1. N. Mostaghel, M. Hejazi and J. Tanbakuchi, 'Response of sliding structure to harmonic support motion', *Earthquake eng. struct. dyn.* **11**, 355–366 (1983).
2. N. Mostaghel and J. Tanbakuchi, 'Response of sliding structures to earthquake support motion', *Earthquake eng. struct. dyn.* **11**, 729–748 (1983).
3. B. Westermo and F. Udwadia, 'Periodic response of a sliding oscillator system to harmonic excitation', *Earthquake eng. struct. dyn.* **11**, 135–146 (1983).
4. M. Iura, K. Matushi and I. Kosaka, 'Analytical expressions for three different modes in harmonic motion of sliding structures', *Earthquake eng. struct. dyn.* **21**, 757–769 (1992).
5. A. Mokha, M. C. Constantinou and A. M. Reinhorn, 'Verification of friction model of Teflon bearings under triaxial load', *J. struct. div. ASCE* **119**, 240–261 (1993).